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Ratio Type Estimator of Population Mean in Double Sampling

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Abstract This paper discusses the problem of estimation of population mean in double sampling. In fact, in this paper double sampling version of (Singh and Tailor, 2003) has been suggested. The bias and mean squared error of the suggested estimators are obtained up to the first degree of approximation. The suggested estimator has been compared with simple mean estimator and double sampling ratio estimator. An empirical study has been carried out to demonstrate the performance of the suggested estimator.
Keywords *Double Sampling, Population, Ratio Type Estimator*

1. Introduction

It is a well known fact that the auxiliary information can increase the efficiency of the estimators of parameters of importance in survey sampling. It is shown that the estimator due to (Kawathekar and Ajgaonkar, 1984) is a member of the proposed class of estimators. Various authors (Kadilar and Cingi, 2004; Jhajj *et al*., 2006; Sodipo and Obisesan, 2007) have defined various estimators including ratio, regression, difference etc. for estimating the unknown population parameters of study variable y, by using the prior knowledge of population mean X of auxiliary variable x, which is highly correlated with study variable y. If the information on population mean X is missing then two phase (double) sampling technique has been generally recommended. In the two-phase sampling scheme, a large preliminary random sample (called first phase sample) is drawn from the population and information on auxiliary variable is taken, which is used to estimate the value of unknown population mean X of auxiliary variable x. Then second phase sample is drawn either from the first phase sample or independently from the population and observations on both study and auxiliary variable are taken. (Housila, 2012) estimated finite population mean in two-phase sampling with known coefficient of variation of an auxiliary character.

2. Materials and Methods

In this paper double sampling version of (Singh and Tailor, 2003) has been suggested.

3. Results and Discussion

Usual procedure of double sampling is described as below:

(i) A large sample S_1 of size $n'(n' < N)$ is drawn and observations are taken only on auxiliary variate to estimate population mean of auxiliary variate;

(ii) Then a sample S_2 of size $n(n < n')$ is drawn either from S_1 (case I) or directly from the population of size N to observe both study variate as well as auxiliary variate.

Let us consider a finite population $U = (U_1, U_2, ..., U_N)$ of size N. Let *y* and *x* be the study and auxiliary variates and y_1 and x_1 be the observations taken on study variate *y* and auxiliary variate *x* respectively. A sample of size *n* is drawn to estimation of the population mean $\hat{\bar{Y}}$ of the study variate *y*.

The classical ratio estimator given by (Cochran, 1940) is defined as,

$$\hat{\bar{Y}}_R = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) \tag{1.1}$$

Where, $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ are unbiased estimators of population mean $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ and $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$ respectively.

When population mean of auxiliary variate $\bar{X}$ is not known, double sampling ratio estimator is defined as,

$$\hat{\bar{Y}}_R^d = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right) \tag{1.2}$$

Where, $\bar{x}' = \frac{1}{n'}\sum_{i=1}^{n'} x_i$ is an unbiased estimator of population mean $\bar{X}$ based on sample of size n'

Thus the bias and mean squared error of $\hat{\bar{Y}}_R^d$ up to the first degree of approximation is obtained as

$$B(\hat{\bar{Y}}_R^d) = \bar{Y} f_3 [C_x^2 (1 - K_{yx})] \tag{1.3}$$

$$MSE(\hat{\bar{Y}}_R^d) = \bar{Y}^2 [f_1 C_y^2 + C_x^2 f_3 (1 - 2\rho_{yx} C_y / C_x)]$$

$$MSE(\hat{\bar{Y}}_R^d) = \bar{Y}^2 [f_1 C_y^2 + C_x^2 f_3 (1 - 2K_{yx})] \tag{1.4}$$

2. Suggested Estimator

(Singh and Tailor, 2003) utilized information on correlation coefficient ρ_{xy} between study variate *y* and auxiliary variate *x* and suggested ratio type estimator as,

$$\hat{\bar{Y}}_{ST} = \bar{y}\left(\frac{\bar{X}+\rho_{yx}}{\bar{x}+\rho_{yx}}\right) \tag{2.1}$$

$\hat{\bar{Y}}_{ST}$ in double sampling is expressed as,

$$\hat{\bar{Y}}_{ST}^{(d)} = \bar{y}\left(\frac{\bar{x}'+\rho_{yx}}{\bar{x}+\rho_{yx}}\right) \tag{2.2}$$

To obtain the bias and mean squared error of the suggested estimator $\hat{\bar{Y}}_{ST}^{(d)}$ we write

$\bar{y} = \bar{Y}(1+e_0), \bar{x} = \bar{X}(1+e_1)$ and $\bar{x}' = \bar{X}(1+e_2)$

Such that $E(e_0) = E(e_1) = E(e_2) = 0$

$E(e_0^2) = f_1 C_y^2$, $E(e_1^2) = f_1 C_x^2$, $E(e_2^2) = f_2 C_x^2$, $E(e_0 e_1) = f_1 \rho_{yx} C_y C_x$

$E(e_0 e_2) = f_2 \rho_{yx} C_y C_x$, $E(e_1 e_2) = f_2 C_x^2$.

Expressing $\hat{\bar{Y}}_{ST}^{(d)}$ in terms of e_i' s we have

$$\hat{\bar{Y}}_{ST}^{(d)} = \bar{Y}(1+e_0)(1+\theta e_2)(1+\theta e_1)^{-1}$$

Thus, bias and mean squared error of $\hat{\bar{Y}}_{ST}^{(d)}$ upto first degree of approximation are obtained as,

$$B(\hat{\bar{Y}}_{ST}^{(d)}) = \bar{Y} f_3 \theta C_x^2 (\theta - K_{yx}) \tag{2.3}$$

$$MSE(\hat{\bar{Y}}_{ST}^{(d)}) = \bar{Y}^2 \left(f_1 C_y^2 + f_3 \theta C_x^2 (\theta - 2\rho_{yx} \frac{C_y}{C_x}) \right) \tag{2.4}$$

$$MSE(\hat{\bar{Y}}_{ST}^{(d)}) = \bar{Y}^2 \left[f_1 C_y^2 + f_3 \theta C_x^2 (\theta - 2K_{yx}) \right] \tag{2.5}$$

3. Efficiency Comparisons

It is well known under simple random sampling without replacement (SRSWOR) that

$$V(\bar{y}) = f_1 \bar{Y}^2 C_y^2 \tag{3.1}$$

Comparison of (2.4) and (3.1) shows that the suggested estimator $\hat{\bar{Y}}_{ST}^{(d)}$ would be more efficient than simple mean estimator $\bar{y}$ i.e.

$$MSE(\hat{\bar{Y}}_{ST}^{(d)}) - V(\bar{y}) < 0 \quad if \;\; 0 < \theta < 2K_{yx} \tag{3.2}$$

Comparing of (1.4) and (2.4), it is observed that the suggested estimator $\hat{\bar{Y}}_{ST}^{(d)}$ would more efficient than ratio estimator in double sampling $\hat{\bar{Y}}_R^{(d)}$ i.e.

$$MSE(\hat{\bar{Y}}_{ST}^{(d)}) - MSE(\hat{\bar{Y}}_R^{d}) < 0 \;\; if \left. \begin{array}{ll} either & 2k-1 < \theta < 1 \\ & \\ or & 1 < \theta < 2k-1 \end{array} \right\} \tag{3.3}$$

4. Empirical Study

To see the performance of the suggested estimator $\hat{\bar{Y}}_{ST}^{d}$ over simple mean estimator $\bar{y}$ and double sampling ratio estimator $\hat{\bar{Y}}_R^{d}$ two natural population data sets are being considered. Descriptions of the population are given below:

Population I: [**Source:** (Das, 1988)]

X: the number of agricultural laboures for 1961,
Y: the number of agricultural laboures for 1971,

$\bar{X}$ = 25.1110, $\bar{Y}$ = 39.0680, N = 278, n = 60, n' = 180
C_y= 1.4451, C_x = 1.6198, ρ_{yx} = 0.7213

Population II: [**Source:** (Cochran, 1977)]

x: The number of rooms per block
y: The number of persons per block

$\bar{Y}$ = 101.1, $\bar{X}$ = 58.80, C_y = 0.14450
C_x = 0.1281, ρ_{xy} = 0.6500, N = 20, n = 8, n' = 12

Table 1: *Percent Relative Efficiency of* $\bar{y}$, $\hat{\bar{Y}}_{R}^{(d)}$, $\hat{\bar{Y}}_{ST}^{(d)}$ *with Respect* $\bar{y}$

Estimator	**PRE** $(., \bar{y})$	
	Population	
	I	**II**
$\bar{y}$	100	100
$\hat{\bar{Y}}_{R}^{(d)}$	142.11	117.65
$\hat{\bar{Y}}_{ST}^{(d)}$	150	125

Table 1 exhibit that the suggested ratio estimator $\hat{\bar{Y}}_{ST}^{(d)}$ has highest percent relative efficiency in comparison to simple mean estimator $\bar{y}$ and double sampling ratio estimator $\hat{\bar{Y}}_{R}^{(d)}$. Thus suggested estimator $\hat{\bar{Y}}_{ST}^{(d)}$ is recommended for its use in practice for the estimation of population mean.

In order to improve the efficiency of the estimators, auxiliary information is used at both selections as well as estimation stages to improve the efficiency of the estimators. (Cochran, 1940) used auxiliary information at estimation stage and proposed ratio estimator. (Murthy, 1964) envisaged product estimator, (Searls, 1964) used coefficient of variation of study variate, motivated by (Searls, 1964), (Sisodia and Dwivedi, 1981) utilized coefficient of variation of auxiliary variate. (Srivenkataramana, 1980) first proposed dual to ratio estimator. (Singh and Tailor, 2005) and (Tailor and Sharma, 2009) worked on ratio-cum-product estimators. These motivates author to propose a new ratio-cum–dual to ratio estimator utilizing dual to ratio estimator of finite population mean.

5. Conclusion

In this paper, the bias and mean squared error of the suggested estimators are obtained up to the first degree of approximation. The suggested estimator has been compared with simple mean estimator and double sampling ratio estimator. An empirical study has been carried out to demonstrate the performance of the suggested estimator.

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