Cloud Publications
International Journal of Advanced Mathematics and Statistics
2016, Volume 2, Issue 1, pp. 52-57, Article ID Sci-490
ISSN 2348-5175

Research Article **Open Access**

An Approach to Construct a Control Chart for Standard Deviation Based on Six Sigma

Radhakrishnan R.[1] and Balamurugan P.[2]

[1]Maharaja Group of Institutions, Perundurai, Tamilnadu
[2]Department of Statistics, Government Arts College (Autonomous), Salem, Tamilnadu, India

Publication Date: 13 September 2016

Article Link: http://scientific.cloud-journals.com/index.php/IJAMS/article/view/Sci-490

Abstract In present scenario companies started applying six sigma concept in their manufacturing process, which results in lesser number of defects compared with existing Shewhart control chart. In this article an attempt is made to construct a control chart based on standard deviation with six sigma especially planned for companies applying this technique in their business and constructed table also presented for the experts to take quick decisions.
Keywords *Control Chart; Process Control; Six Sigma; Six Sigma Quality Level*

1. Introduction

The control charts suggested by W.A. Shewhart (1931) was based on 3 sigma control limits. Radhakrishnan (2009a, 2009b, 200c, 2010a, 2010b, 2010c, 2011 and 2012) described as if the same charts are used for the products of the companies which adopt Six sigma concept in their process, and then no point will fall outside the control limits because of the improvement in the quality. So a separate control chart is necessary to check the results of the companies, which adopt Six sigma. The practice was aimed at taking action to improve the overall performance and the companies, which are practicing Six sigma, are expected to produce 3.4 or less number of defects per million opportunities. In this research an effort is made to construct a control chart based on the concept Six sigma for Standard deviation and the Table 1 is also provided for the experts to build rapid judgments on the floor itself.

2. Concepts and Terminologies

Upper Specification Limit (USL)

It is the greatest amount specified by the producer for a process or product to have the acceptable performance.

Lower Specification Limit (LSL)

It is the smallest amount specified by the producer for a process or product to have the acceptable performance.

Tolerance Level (TL)

It is the difference between USL and LSL, TL = USL-LSL

Process Capability (Cp)

This is the ratio of tolerance level to six times standard deviation of the process.

$$c_p = (TL/6\sigma) = (USL - LSL/6\sigma)$$

Standard Deviation (σ)

For many purposes standard deviation is the most useful measure of dispersion of a set of numbers. It is the root mean square value.

Quality Control Constant[1] ($O_{6\sigma}$)

The constant $O_{6\sigma}$ introduced in this paper to determine the control limits based on six sigma initiatives for standard deviation.

Quality Control Constant[2] (c_4)

The constant c_4 introduced in this paper to determine the control limits based on 3sigma for standard deviation.

3. Construction of Control Chart Based on Six Sigma Initiatives for Standard Deviation

Fix the tolerance level (TL) and process capability (C_P) to determine the process standard deviation ($\sigma_{6\sigma}$). Apply the value of $\sigma_{6\sigma}$ in the control limits $\bar{S} \pm \left\{\left(O_{6\sigma} \times \sqrt{1-c_4^{\,2}}\right) \times \sigma_{6\sigma}\right\}$, to get the control limits based on six sigma initiatives for Standard deviation. The value of $O_{6\sigma}$ is obtained using $p(z \le z_{6\sigma}) = 1-\alpha_1, \alpha_1 = 3.4 \text{ x } 10^{-6}$ and z is a standard normal variate. For a specified TL and C_P of the process, the value of σ (termed as $\sigma_{6\sigma}$) is calculated from $c_p = \frac{TL}{6\sigma}$ using a C program and presented in Table 1.2 for various combinations of TL and C_P. The control limits based on six sigma initiatives for Standard deviation chart are

$$UCL_{6\sigma} = \bar{S} + \left\{\left(O_{6\sigma} \times \sqrt{1-c_4^{\,2}}\right)\right\} \times \sigma_{6\sigma}$$

$$Center\ line\ CL_{6\sigma} = \bar{S}$$

$$LCL_{6\sigma} = \bar{S} - \left\{\left(O_{6\sigma} \times \sqrt{1-c_4^{\,2}}\right)\right\} \times \sigma_{6\sigma}$$

Table 1: *Values for a specified Cp and TL*

TL / C_p	0.0131	0.0132	0.0133	0.0134	0.0135
1.0	0.0022	0.0022	0.0022	0.0022	0.0023
1.1	0.0020	0.0020	0.0020	0.0020	0.0020
1.2	0.0018	0.0018	0.0018	0.0019	0.0019
1.3	0.0017	0.0017	0.0017	0.0017	0.0017
1.4	0.0016	0.0016	0.0016	0.0016	0.0016
1.5	0.0015	0.0015	0.0015	0.0015	0.0015
1.6	0.0014	0.0014	0.0014	0.0014	0.0014
1.7	0.0013	0.0013	0.0013	0.0013	0.0013
1.8	0.0012	0.0012	0.0012	0.0012	0.0013
1.9	0.0011	0.0012	0.0012	0.0012	0.0012
2.0	0.0011	0.0011	0.0011	0.0011	0.0011
2.1	0.0010	0.0010	0.0011	0.0011	0.0011
2.2	0.0010	0.0010	0.0010	0.0010	0.0010
2.3	0.0010	0.0010	0.0010	0.0010	0.0010
2.4	0.0009	0.0009	0.0009	0.0093	0.0009
2.5	0.0009	0.0009	0.0090	0.0089	0.0009

4. Conditions for Application

- Human involvement should be less in the manufacturing process
- The company adopts Six sigma quality initiatives in its processes

Example

The following results of inside Diameter Measurement (mm) for Automobile Engine Piston Rings.

Table 2: *Inside Diameter Measurement (mm) for Automobile Engine Piston Rings and Standard deviation (S_i)*

Sample Number			Observation			S_i
1	74.030	74.002	74.019	73.992	74.008	0.0148
2	73.995	73.992	74.001	74.011	74.004	0.0075
3	73.988	74.024	74.021	74.005	74.002	0.0147
4	74.002	73.996	73.993	74.015	74.009	0.0091
5	73.992	74.007	74.015	73.989	74.014	0.0122
6	74.009	73.994	73.997	73.985	73.993	0.0087
7	73.995	74.006	73.994	74.000	74.005	0.0055
8	73.985	74.003	73.993	74.015	73.988	0.0123
9	74.008	73.995	74.009	74.005	74.004	0.0055
10	73.998	74.000	73.990	74.007	73.995	0.0063
11	73.994	73.998	73.994	73.995	73.990	0.0029
12	74.004	74.000	74.007	74.000	73.996	0.0042
13	73.983	74.002	73.998	73.997	74.012	0.0105
14	74.006	73.967	73.994	74.000	73.984	0.0153
15	74.012	74.014	73.998	73.999	74.007	0.0073
16	74.000	73.984	74.005	73.998	73.996	0.0078
17	73.994	74.012	73.986	74.005	74.007	0.0106
18	74.006	74.010	74.018	74.003	74.000	0.0070
19	73.984	74.002	74.003	74.005	73.997	0.0085

20	74.000	74.010	74.013	74.020	74.003	0.0080
21	74.982	74.001	74.015	74.005	73.996	0.0122
22	74.004	73.999	73.990	74.006	74.009	0.0074
23	74.010	73.989	73.990	74.009	74.014	0.0119
24	74.015	74.008	73.993	74.000	74.010	0.0087
25	73.982	73.984	73.995	74.017	74.013	0.0162

n = 5 and $\bar{S}$ = 0.0094

4.1a. Three Sigma Control Limits for Standard Deviation Chart

The 3σ control limits suggested by Shewhart (1931) are $\bar{S} \pm \left\{\left(3\sqrt{1-c_4^{\,2}}\right)\left(\bar{S}/c_4\right)\right\}$

For n = 4, $c_4 = 0.9400$ (Quality control factor, W.A.Shewhart)

$$\mathrm{UCL}_{3\sigma} = \bar{S} + \left\{\left(3\sqrt{1-c_4^{\,2}}\right)\left(\bar{S}/c_4\right)\right\} = 0.0094 + \left\{\left(3\sqrt{1-0.9400^2}\right)(0.0094/0.9400)\right\}$$
$$= 0.0094 + 0.0102 = 0.0196$$
$$\text{Central Line } \mathrm{CL}_{3\sigma} = \bar{S} = 0.0094$$
$$\mathrm{LCL}_{3\sigma} = \bar{S} - \left\{\left(3\sqrt{1-c_4^{\,2}}\right)\left(\bar{S}/c_4\right)\right\} = 0.0094 - \left\{\left(3\sqrt{1-0.9400^2}\right)(0.0094/0.9400)\right\}$$
$$= 0.0094 - 0.0102 = 0$$

From the resulting Figure 1 that the process is in control, since all the samples lie inside the control limits.

4.1b. Control Limits Based on Six Sigma Initiatives for Standard Deviation Chart

For a given TL = 0.0133 (USL-LSL =0.0162-0.0029) & C_p = 1.5, it is found from the Table 1 that the value of $\sigma_{6\sigma}$ is 0.0015. The control limits based on six sigma initiatives for Standard deviation chart for a specified TL and $O_{6\sigma}$ are $\bar{S} \pm \left\{\left(4.831 \times \sqrt{1-c_4^{\,2}}\right) \times \sigma_{6\sigma}\right\}$ with

$$UCL_{6\sigma} = \bar{S} + \left\{\left(O_{6\sigma} \times \sqrt{1-c_4^{\,2}}\right)\right\} \times \sigma_{6\sigma} = 0.0094 + \left\{\left(4.831 \times \sqrt{1-0.9400^2}\right) \times 0.0015\right\} = 0.0119$$
$$Center\ line\ CL_{6\sigma} = \bar{S} = 0.0094$$
$$LCL_{6\sigma} = \bar{S} - \left\{\left(O_{6\sigma} \times \sqrt{1-c_4^{\,2}}\right)\right\} \times \sigma_{6\sigma} = 0.0094 - \left\{\left(4.831 \times \sqrt{1-0.9400^2}\right) \times 0.0015\right\} = 0.0069$$

From the resulting Figure 1 that the sample numbers 1, 3, 5, 8, 14, 21 and 25 goes above the upper control limit and the sample numbers 7, 9, 10, 11 and 12 goes below the lower control limit. Therefore the process does not exhibit statistical control.

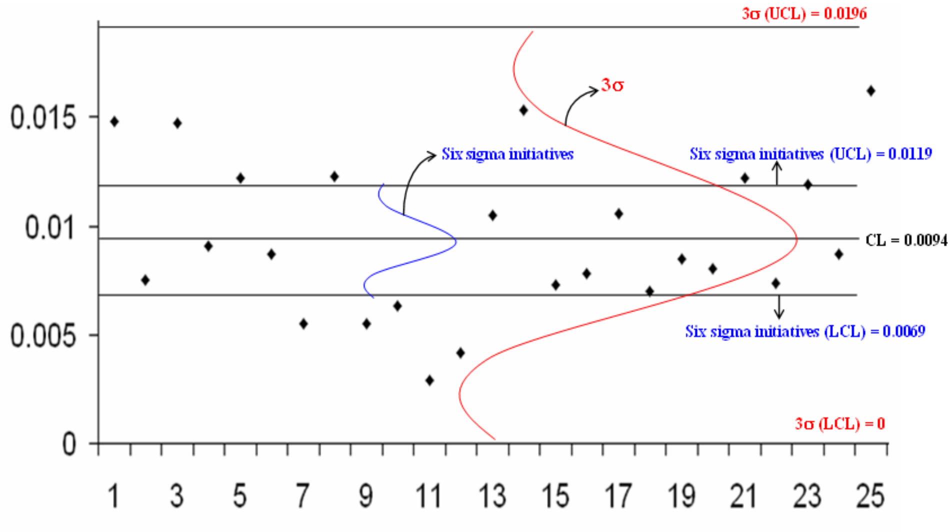

Figure 1: *Comparison of the process: 3σ limits and control limits using Six Sigma initiatives*

5. Conclusion

In this paper, a procedure is given to construct a control chart based on six sigma initiatives for Standard deviation with an example. It is found that the process was not in control even when Six Sigma initiatives are adopted. It is very clear from the comparison that when the process is centered with reduced variation many points fall outside the control limits than the 3 sigma control limits, which indicate that the process is not in the level it was expected. So a correction in the process is very much required to reduce the variations. In future, all the companies are adopting this technique instead of the existing Shewhart chart in their organization.

References

Radhakrishnan, R., 2009a: *Construction of Six Sigma Based Sampling Plans.* A D.Sc. Thesis Submitted to Bharathiar University, Coimbatore, India.

Radhakrishnan, R. and Balamurugan, P., 2009b: Six Sigma Based Attribute Control Charts for Defects. Proceedings of the 2009 International Marketing in Asia Pacific: Issues and Challenges, GRD School of Commerce and International Business, Coimbatore, Tamilnadu, India and Jointly Organized by University of central Lancashire, Nov. 4-5. 348-353.

Radhakrishnan, R. and Balamurugan, P., 2009c: Construction of Control Charts Based On Six Sigma Initiatives. Proceedings of the 2009 International Conference on Trends in Information Technology and Business Intelligence (ITBI 2009), Institute of Management Technology, Nagpur, India. Nov 4-6, 356-361.

Radhakrishnan, R. and Balamurugan, P., 2010a: Six Sigma based Control Charts for the Number of Defectives. Proceedings of the 2010 International Conference on Industrial Engineering and Operations Management (IEOM 2010) organized by Bangladesh Society of Mechanical Engineers, Dhaka, Bangladesh, Jan. 9-10. 92.

Radhakrishnan, R. and Balamurugan, P. 2010b: Six Sigma based Exponentially Weighted Moving Average Control Chart. *Indian Journal of Science and Technology*. 9-10 (3) 1052-1055.

Radhakrishnan, R. and Balamurugan, P. 2010c: Six Sigma based Control Chart for Fraction Defectives. *Journal of Testing and Evaluation*. 39 (4) 678-681.

Radhakrishnan, R. and Balamurugan, P. 2011: Construction of Control Charts Based on Six Sigma Initiatives for the Number of Defects and Average Number of Defects per unit. *Journal of Modern Applied Statistical Methods*.

Radhakrishnan, R. and Balamurugan, P. 2012: Construction of Control Charts Based on Six Sigma Initiatives for Fraction Defectives with Varying Sample Size. *Journal of Statistics & Management Systems (JSMS)*.

Shewhart, W.A., 1931: *Economic Control of Quality of Manufactured Product.* Van Nostrand, New York.